Extrema on an Interval

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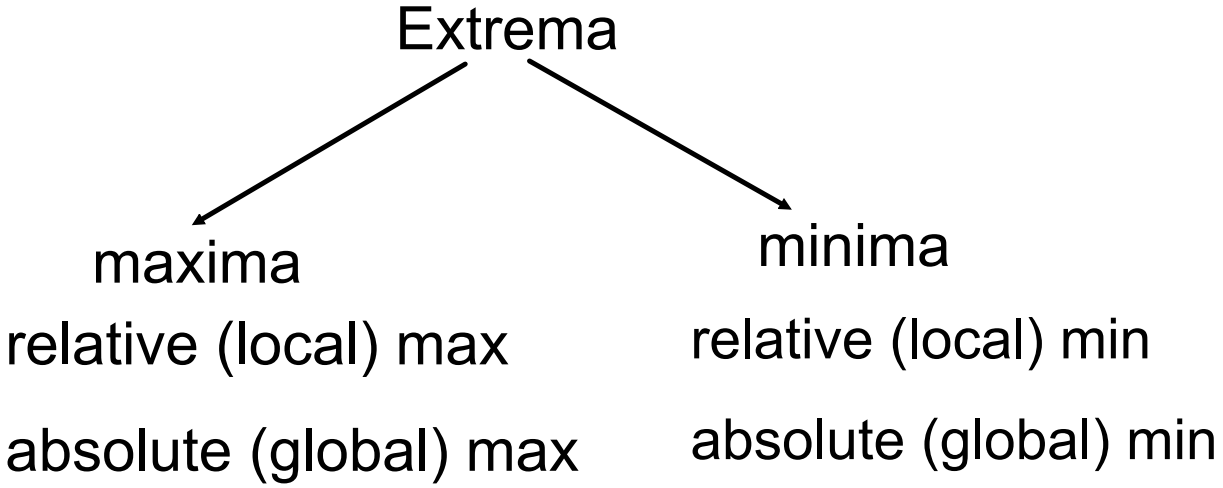
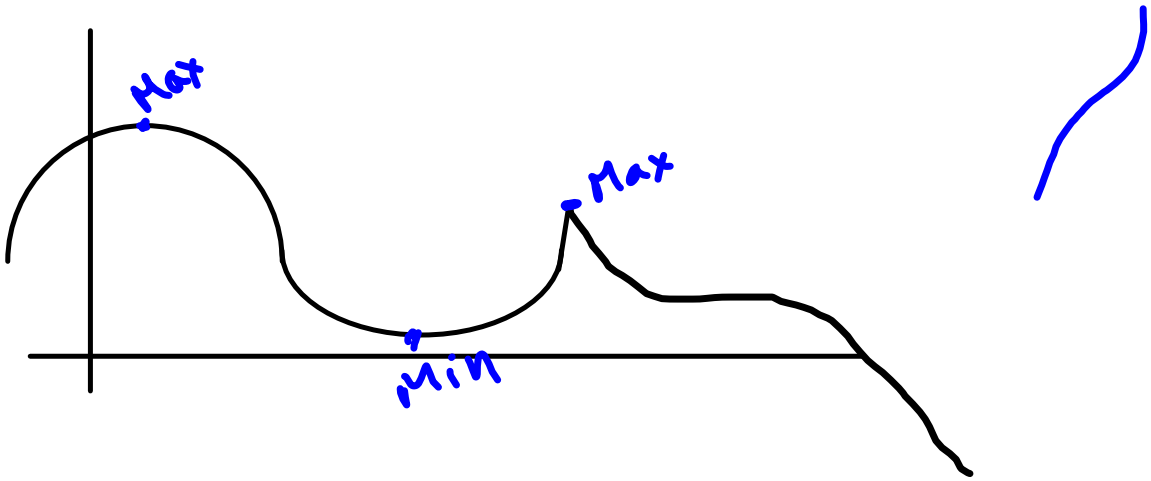
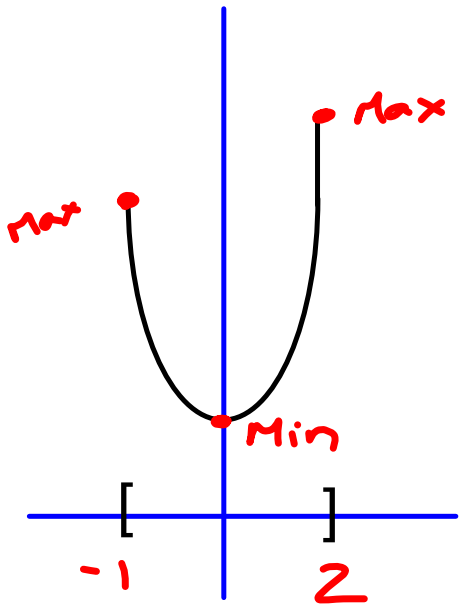
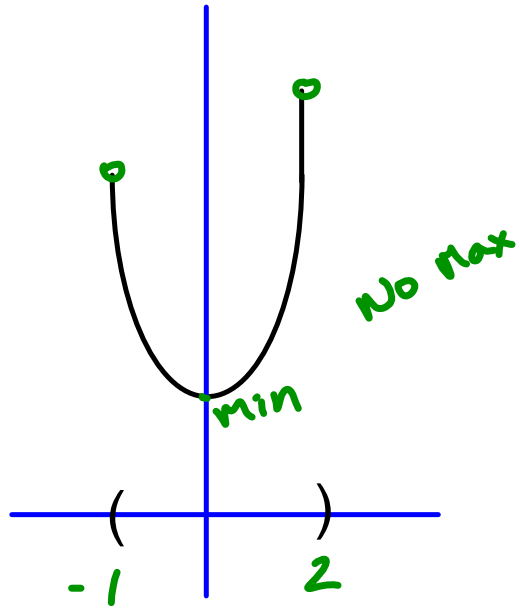

Extrema
maxima
minima
relative (local) max
absolute (global) max
relative (local) min
absolute (global) min

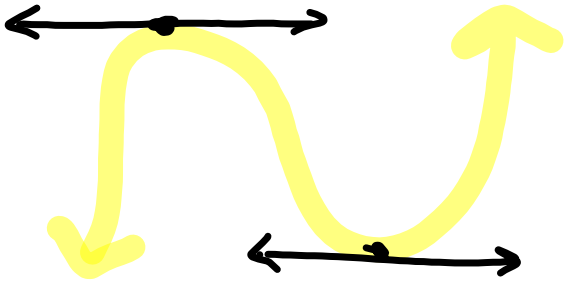

Max
Max
Min

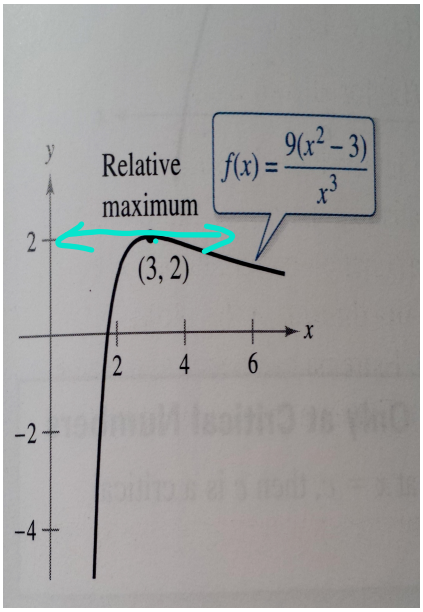

max
max
Min
-1
2


No max
min
-1
2

Extreme Value Thm

If *f* is continuous on a closed interval [a,b], then *f* has both a minimum and a maximum on the interval.





$$f(x) = \frac{9(x^2-3)}{x^3}$$

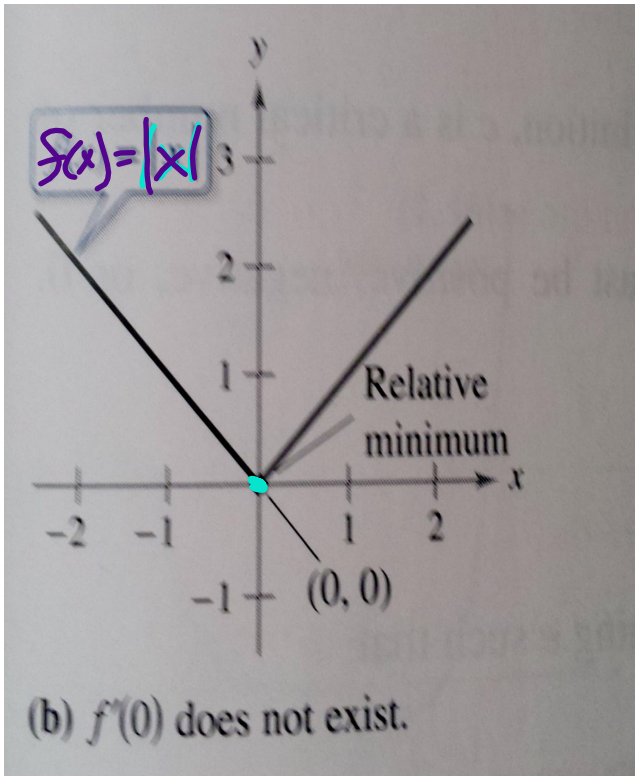
$$f'(x) = \frac{18x(x^3) - 9(x^2-3)(3x^2)}{(x^3)^2}$$ ① Derivative

$$f'(x) = \frac{9(9-x^2)}{x^4}$$

$f'(3) = 0$

② find slope at (3, 2)

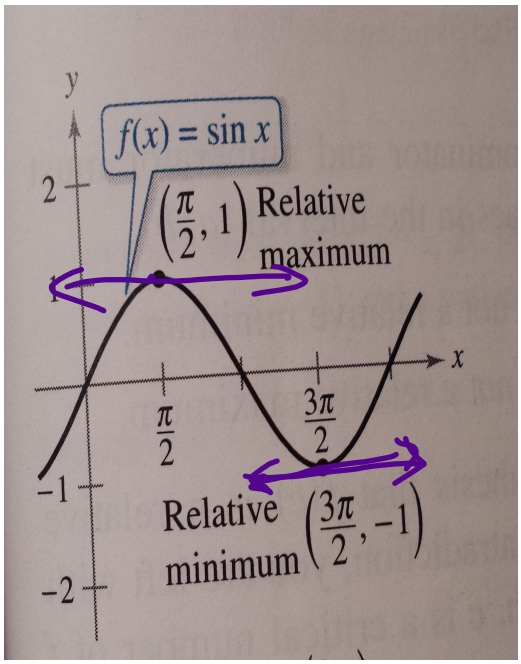
Horizontal Tangent line



(b) $f'(0)$ does not exist.

$f'(x) = DNE$

Relative extrema occur when there's a horizontal tangent line or the $f'(x) = DNE$



$f(x) = \sin x$

$f'(x) = \cos x$

$f'\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$

$f'\left(\frac{3\pi}{2}\right) = \cos\frac{3\pi}{2} = 0$

The derivative of a relative extremum is either zero or DNE.

The x-value of the relative extremum is called a...

Critical Number

Let f be defined at c.

If $f'(c)=0$ or if *f* is not differentiable at c, then c is a critical number of *f*.

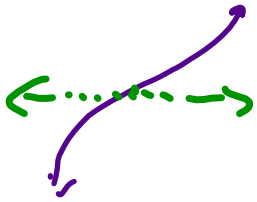
Relative extrema only occur at critical numbers.

open interval

If f has a relative min or relative max at x=c, then c is a critical number.

Not all critical #'s are max/min

Not all horizontal tan lines are max/Min



Find the extrema of

$$f(x) = 3x^4 - 4x^3 \qquad [-1,2]$$

① Derivative

$f'(x) = 12x^3 - 12x^2$

② find CN

$f'(x) = 0$

Solve for x

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$12x^2 = 0 \quad x-1 = 0$

$x = 0 \quad x = 1$

③ list CN & endpoints

④ Plug all # into orig f(x)

-1	0	1	2
f(-1)	f(0)	f(1)	f(2)
7	0	-1	16
		Min.	Max

2 sinx - cos 2x $[0, 2\pi]$

$f'(x) = 2\cos x + 2\sin 2x$ $\quad \sin 2x = 2\sin x \cos x$

$f'(x) = 2\cos x + 4\sin x \cos x$

$f'(x) = 2\cos x (1 + 2\sin x) = 0$

$2\cos x = 0$ $\quad 1 + 2\sin x = 0$

$\cos x = 0$ $\quad \sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad x = \frac{7\pi}{6}, \frac{11\pi}{6}$

0 $f(0)$	$\pi/2$ $f(\pi/2)$	$3\pi/2$ $f(3\pi/2)$	$7\pi/6$ $f(7\pi/6)$	$11\pi/6$ $f(11\pi/6)$	2π $f(2\pi)$
-1	3	-1	$-3/2$	$-3/2$	-1
	Max		Min	Min	